Faith, Evolution, and Programming Languages

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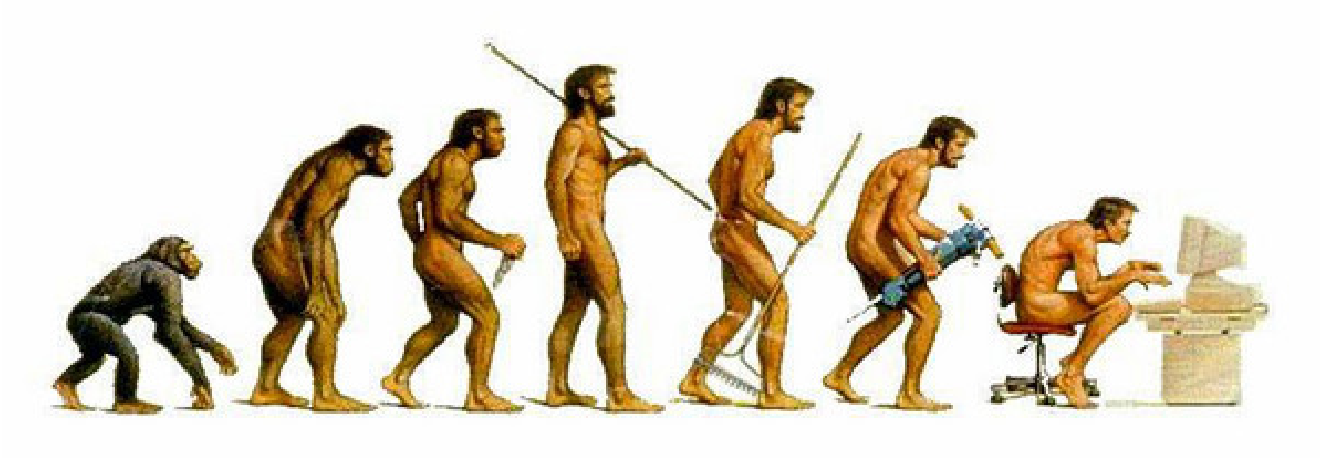
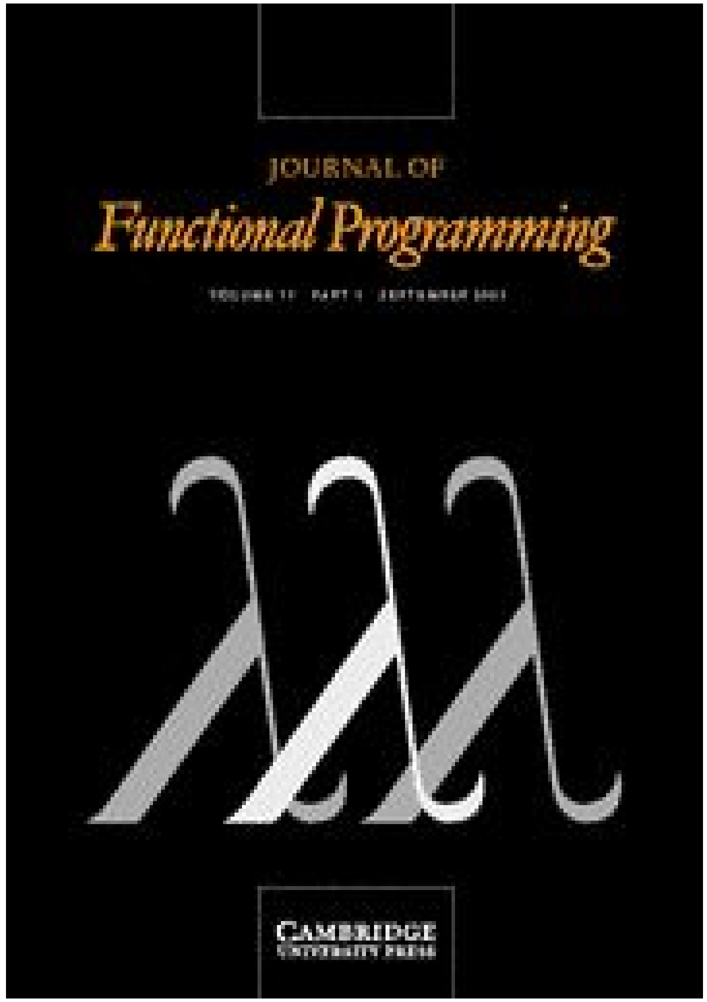


Evolution

Multiculturalism

Part I

Church: The origins of faith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Gerhard Gentzen (1909–1945)

Gerhard Gentzen (1935) — Natural Deduction

$\supset\!-I$

$$\frac{\begin{matrix}[\mathfrak{A}]\\ \mathfrak{B}\end{matrix}}{\mathfrak{A} \supset \mathfrak{B}}$$

$\supset\!-E$

$$\frac{\mathfrak{A} \qquad \mathfrak{A} \supset \mathfrak{B}}{\mathfrak{B}}$$

$\&\!-I$

$$\frac{\mathfrak{A} \qquad \mathfrak{B}}{\mathfrak{A} \,\&\, \mathfrak{B}}$$

$\&\!-E$

$$\frac{\mathfrak{A} \,\&\, \mathfrak{B}}{\mathfrak{A}} \qquad \frac{\mathfrak{A} \,\&\, \mathfrak{B}}{\mathfrak{B}}$$

Gerhard Gentzen (1935) — Natural Deduction

&–*I*

$$\frac{\mathfrak{A} \quad \mathfrak{B}}{\mathfrak{A} \,\&\, \mathfrak{B}}$$

&–*E*

$$\frac{\mathfrak{A} \,\&\, \mathfrak{B}}{\mathfrak{A}} \qquad \frac{\mathfrak{A} \,\&\, \mathfrak{B}}{\mathfrak{B}}$$

∨–*I*

$$\frac{\mathfrak{A}}{\mathfrak{A} \vee \mathfrak{B}} \qquad \frac{\mathfrak{B}}{\mathfrak{A} \vee \mathfrak{B}}$$

∨–*E*

$$\frac{\mathfrak{A} \vee \mathfrak{B} \quad \begin{matrix}[\mathfrak{A}] \\ \mathfrak{C}\end{matrix} \quad \begin{matrix}[\mathfrak{B}] \\ \mathfrak{C}\end{matrix}}{\mathfrak{C}}$$

∀–*I*

$$\frac{\mathfrak{F}a}{\forall \mathfrak{x}\, \mathfrak{F}\mathfrak{x}}$$

∀–*E*

$$\frac{\forall \mathfrak{x}\, \mathfrak{F}\mathfrak{x}}{\mathfrak{F}a}$$

∃–*I*

$$\frac{\mathfrak{F}a}{\exists \mathfrak{x}\, \mathfrak{F}\mathfrak{x}}$$

∃–*E*

$$\frac{\exists \mathfrak{x}\, \mathfrak{F}\mathfrak{x} \quad \begin{matrix}[\mathfrak{F}a] \\ \mathfrak{C}\end{matrix}}{\mathfrak{C}}$$

⊃–*I*

$$\frac{\begin{matrix}[\mathfrak{A}] \\ \mathfrak{B}\end{matrix}}{\mathfrak{A} \supset \mathfrak{B}}$$

⊃–*E*

$$\frac{\mathfrak{A} \quad \mathfrak{A} \supset \mathfrak{B}}{\mathfrak{B}}$$

¬–*I*

$$\frac{\begin{matrix}[\mathfrak{A}] \\ \curlywedge\end{matrix}}{\neg \mathfrak{A}}$$

¬–*E*

$$\frac{\mathfrak{A} \quad \neg \mathfrak{A}}{\curlywedge} \qquad \frac{\curlywedge}{\mathfrak{D}} \;.$$

Gerhard Gentzen (1935) — Natural Deduction

$$\dfrac{\begin{array}{c}[A]^x \\ \vdots \\ B\end{array}}{A \supset B}\ \supset\text{-}\mathbf{I}^x \qquad \dfrac{A \supset B \qquad A}{B}\ \supset\text{-}\mathbf{E}$$

$$\dfrac{A \qquad B}{A \mathbin{\&} B}\ \&\text{-}\mathbf{I} \qquad \dfrac{A \mathbin{\&} B}{A}\ \&\text{-}\mathbf{E}_0 \qquad \dfrac{A \mathbin{\&} B}{B}\ \&\text{-}\mathbf{E}_1$$

Simplifying a proof

$$\dfrac{\dfrac{\dfrac{\dfrac{[B \mathbin{\&} A]^z}{A}\,\&\text{-E}_1 \qquad \dfrac{[B \mathbin{\&} A]^z}{B}\,\&\text{-E}_0}{A \mathbin{\&} B}\,\&\text{-I}}{(B \mathbin{\&} A) \supset (A \mathbin{\&} B)}\,\supset\text{-I}^z \qquad \dfrac{[B]^y \quad [A]^x}{B \mathbin{\&} A}\,\&\text{-I}}{A \mathbin{\&} B}\,\supset\text{-E}$$

Simplifying a proof

$$\dfrac{\dfrac{\dfrac{\dfrac{[B \mathbin{\&} A]^z}{A}\,\&\text{-E}_1 \qquad \dfrac{[B \mathbin{\&} A]^z}{B}\,\&\text{-E}_0}{A \mathbin{\&} B}\,\&\text{-I}}{(B \mathbin{\&} A) \supset (A \mathbin{\&} B)}\,\supset\text{-I}^z \qquad \dfrac{[B]^y \qquad [A]^x}{B \mathbin{\&} A}\,\&\text{-I}}{A \mathbin{\&} B}\,\supset\text{-E}$$

$$\Downarrow$$

$$\dfrac{\dfrac{\dfrac{[B]^y \qquad [A]^x}{B \mathbin{\&} A}\,\&\text{-I}}{A}\,\&\text{-E}_1 \qquad \dfrac{\dfrac{[B]^y \qquad [A]^x}{B \mathbin{\&} A}\,\&\text{-I}}{B}\,\&\text{-E}_0}{A \mathbin{\&} B}\,\&\text{-I}$$

Simplifying a proof

$$\dfrac{\dfrac{\dfrac{\dfrac{[B \mathbin{\&} A]^z}{A}\,\&\text{-E}_1 \qquad \dfrac{[B \mathbin{\&} A]^z}{B}\,\&\text{-E}_0}{A \mathbin{\&} B}\,\&\text{-I}}{(B \mathbin{\&} A) \supset (A \mathbin{\&} B)}\,\supset\text{-I}^z \qquad \dfrac{[B]^y \quad [A]^x}{B \mathbin{\&} A}\,\&\text{-I}}{A \mathbin{\&} B}\,\supset\text{-E}$$

$$\Downarrow$$

$$\dfrac{\dfrac{\dfrac{[B]^y \quad [A]^x}{B \mathbin{\&} A}\,\&\text{-I}}{A}\,\&\text{-E}_1 \qquad \dfrac{\dfrac{[B]^y \quad [A]^x}{B \mathbin{\&} A}\,\&\text{-I}}{B}\,\&\text{-E}_0}{A \mathbin{\&} B}\,\&\text{-I}$$

$$\Downarrow$$

$$\dfrac{[A]^x \quad [B]^y}{A \mathbin{\&} B}\,\&\text{-I}$$

Alonzo Church (1903–1995)

Alonzo Church (1932) — Lambda calculus

An occurrence of a variable **x** in a given formula is called an occurrence of **x** as a *bound variable* in the given formula if it is an occurrence of **x** in a part of the formula of the form $\lambda \mathbf{x}[\mathbf{M}]$; that is, if there is a formula **M** such that $\lambda \mathbf{x}[\mathbf{M}]$ occurs in the given formula and the occurrence of **x** in question is an occurrence in $\lambda \mathbf{x}[\mathbf{M}]$. All other occurrences of a variable in a formula are called occurrences as a *free variable.*

A formula is said to be *well-formed* if it is a variable, or if it is one

Alonzo Church (1940) — Typed λ-calculus

$$\dfrac{\begin{array}{c}[x : A]^x \\ \vdots \\ u : B\end{array}}{\lambda x.\, u : A \supset B} \supset\text{-I}^x \qquad \dfrac{s : A \supset B \qquad t : A}{s\, t : B} \supset\text{-E}$$

$$\dfrac{t : A \qquad u : B}{\langle t, u\rangle : A \,\&\, B} \,\&\text{-I} \qquad \dfrac{s : A \,\&\, B}{s_0 : A} \,\&\text{-E}_0 \qquad \dfrac{s : A \,\&\, B}{s_1 : B} \,\&\text{-E}_1$$

Simplifying a program

$$\dfrac{\dfrac{\dfrac{\dfrac{[z : B \,\&\, A]^z}{z_1 : A}\,\&\text{-E}_1 \qquad \dfrac{[z : B \,\&\, A]^z}{z_0 : B}\,\&\text{-E}_0}{\langle z_1, z_0\rangle : A \,\&\, B}\,\&\text{-I}}{\lambda z.\,\langle z_1, z_0\rangle : (B \,\&\, A) \supset (A \,\&\, B)}\,\supset\text{-I}^z \qquad \dfrac{[y : B]^y \qquad [x : A]^x}{\langle y, x\rangle : B \,\&\, A}\,\&\text{-I}}{(\lambda z.\,\langle z_1, z_0\rangle)\,\langle y, x\rangle : A \,\&\, B}\,\supset\text{-E}$$

Simplifying a program

$$
\dfrac{\dfrac{\dfrac{\dfrac{[z : B \mathbin{\&} A]^z}{z_1 : A}\,\&\text{-E}_1 \qquad \dfrac{[z : B \mathbin{\&} A]^z}{z_0 : B}\,\&\text{-E}_0}{\langle z_1, z_0\rangle : A \mathbin{\&} B}\,\&\text{-I}}{\lambda z.\, \langle z_1, z_0\rangle : (B \mathbin{\&} A) \supset (A \mathbin{\&} B)}\,\supset\text{-I}^z \qquad \dfrac{[y : B]^y \quad [x : A]^x}{\langle y, x\rangle : B \mathbin{\&} A}\,\&\text{-I}}{(\lambda z.\, \langle z_1, z_0\rangle)\,\langle y, x\rangle : A \mathbin{\&} B}\,\supset\text{-E}
$$

$$\Downarrow$$

$$
\dfrac{\dfrac{\dfrac{[y : B]^y \quad [x : A]^x}{\langle y, x\rangle : B \mathbin{\&} A}\,\&\text{-I}}{\langle y, x\rangle_1 : A}\,\&\text{-E}_1 \qquad \dfrac{\dfrac{[y : B]^y \quad [x : A]^x}{\langle y, x\rangle : B \mathbin{\&} A}\,\&\text{-I}}{\langle y, x\rangle_0 : B}\,\&\text{-E}_0}{\langle \langle y, x\rangle_1, \langle y, x\rangle_0\rangle : A \mathbin{\&} B}\,\&\text{-I}
$$

Simplifying a program

$$\dfrac{\dfrac{\dfrac{\dfrac{[z : B \,\&\, A]^z}{z_1 : A}\,\&\text{-E}_1 \qquad \dfrac{[z : B \,\&\, A]^z}{z_0 : B}\,\&\text{-E}_0}{\langle z_1, z_0\rangle : A \,\&\, B}\,\&\text{-I}}{\lambda z.\,\langle z_1, z_0\rangle : (B \,\&\, A) \supset (A \,\&\, B)}\supset\text{-I}^z \qquad \dfrac{[y : B]^y \quad [x : A]^x}{\langle y, x\rangle : B \,\&\, A}\,\&\text{-I}}{(\lambda z.\,\langle z_1, z_0\rangle)\,\langle y, x\rangle : A \,\&\, B}\supset\text{-E}$$

$$\Downarrow$$

$$\dfrac{\dfrac{\dfrac{[y : B]^y \quad [x : A]^x}{\langle y, x\rangle : B \,\&\, A}\,\&\text{-I}}{\langle y, x\rangle_1 : A}\,\&\text{-E}_1 \qquad \dfrac{\dfrac{[y : B]^y \quad [x : A]^x}{\langle y, x\rangle : B \,\&\, A}\,\&\text{-I}}{\langle y, x\rangle_0 : B}\,\&\text{-E}_0}{\langle \langle y, x\rangle_1, \langle y, x\rangle_0\rangle : A \,\&\, B}\,\&\text{-I}$$

$$\Downarrow$$

$$\dfrac{[x : A]^x \quad [y : B]^y}{\langle x, y\rangle : A \,\&\, B}\,\&\text{-I}$$

William Howard (1980) — Curry-Howard Isomorphism

THE FORMULAE-AS-TYPES NOTION OF CONSTRUCTION

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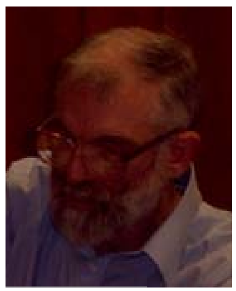
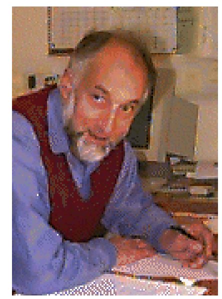
Dedicated to H. B. Curry on the occasion of his 80th birthday.

The following consists of notes which were privately circulated in 1969. Since they have been referred to a few times in the literature, it seems worth while to publish them. They have been rearranged for easier reading, and some inessential corrections have been made.

Curry-Howard



Hindley-Milner



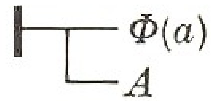
Girard-Reynolds

Part II

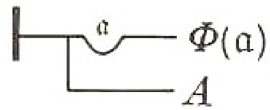
Second-order logic,
Polymorphism,
and Java

Gottlob Frege (1879) — Quantifiers (∀)

It is clear also that from



we can derive



if A is an expression in which a does not occur and if a stands only in the argument places of $\Phi(a)$.[14] If $-\!\overset{\mathfrak{a}}{\smile}\!-\Phi(\mathfrak{a})$ is denied, we must be able to specify a meaning for a such that $\Phi(a)$ will be denied. If, therefore, $-\!\overset{\mathfrak{a}}{\smile}\!-\Phi(\mathfrak{a})$ were to be denied and

John Reynolds (1974) — Polymorphism

TOWARDS A THEORY OF TYPE STRUCTURE †

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Introduction

The type structure of programming languages has been the subject of an active development characterized by continued controversy over basic principles.[1-7] In this paper, we formalize a view of these principles somewhat similar to that of J. H. Morris.[5] We introduce an extension of the typed lambda calculus which permits user-defined types and polymorphic functions, and show that the semantics of this language satisfies a representation theorem which embodies our notion of a "correct" type structure.

Syntax

To formalize the syntax of our language, we begin with two disjoint, countably infinite sets: the set T of type variables and the set V of normal variables. Then W, the set of type expressions, is the minimal set satisfying:

(1a) If $t \in T$ then:

$t \in W$.

(1b) If $w_1, w_2 \in W$ then:

$(w_1 \rightarrow w_2) \in W$.

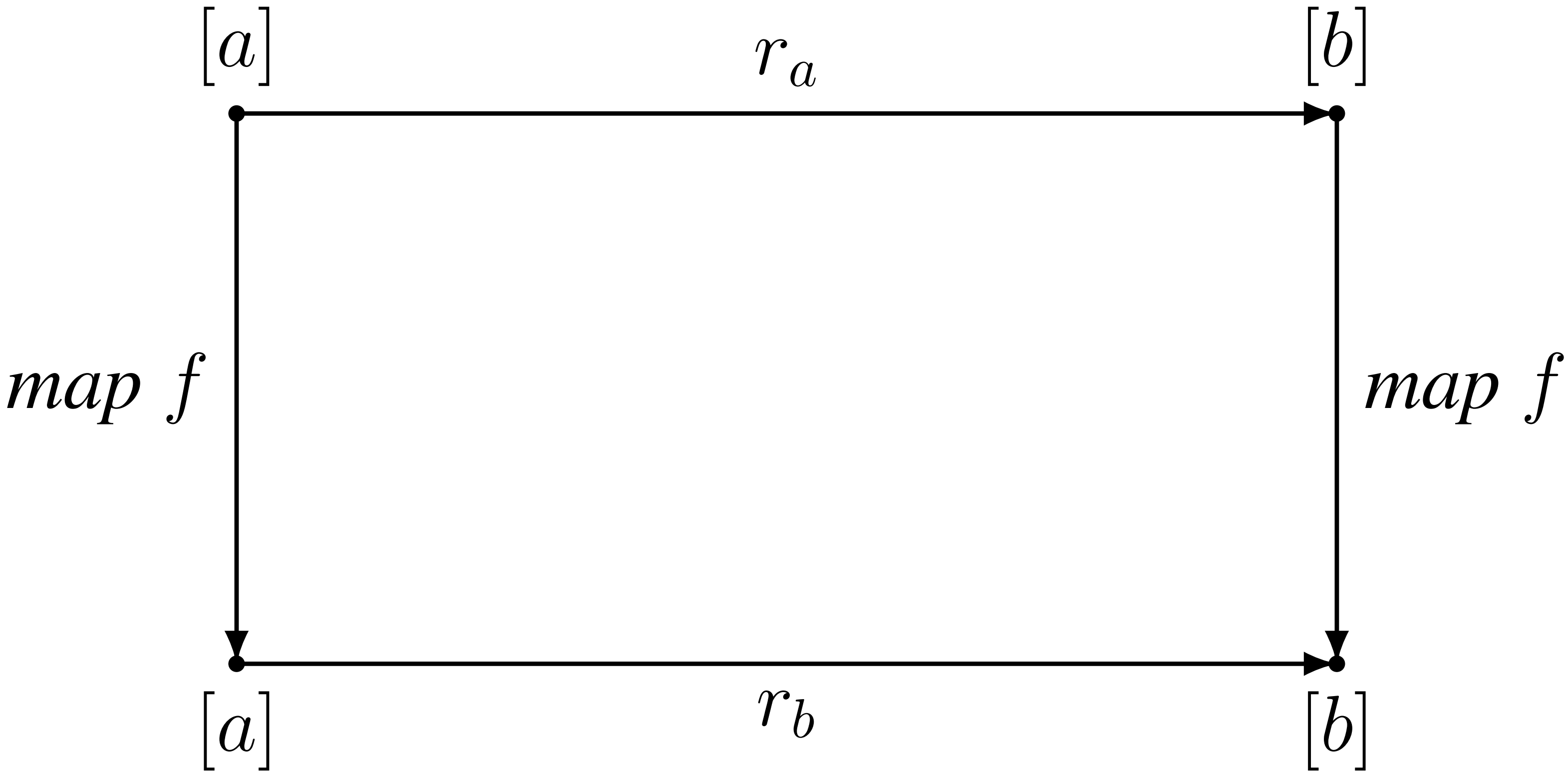
(1c) If $t \in T$ and $w \in W$ then:

$(\Delta t.\ w) \in W$.

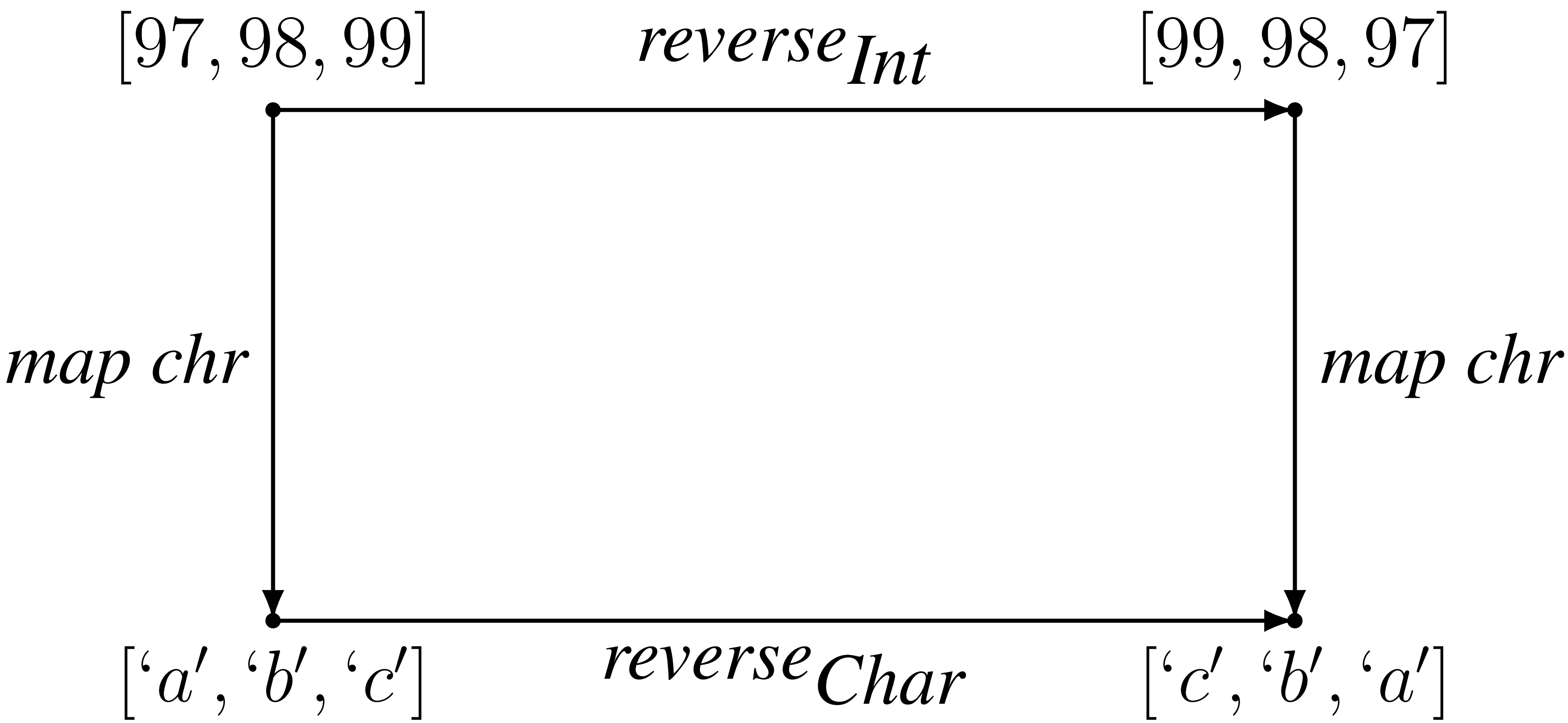
A magic trick

$$r :: [a] \rightarrow [a]$$

Theorems for Free!



Theorems for Free!



Odersky and Wadler (1997) — Pizza

Pizza into Java: Translating theory into practice

Martin Odersky
University of Karlsruhe

Philip Wadler
University of Glasgow

Example 2.1 Polymorphism in Pizza

```
class Pair<elem> {
    elem x;  elem y;
    Pair (elem x, elem y) {this.x = x; this.y = y;}
    void swap () {elem t = x; x = y; y = t;}
}

Pair<String> p = new Pair("world!", "Hello,");
p.swap();
System.out.println(p.x + p.y);

Pair<int> q = new Pair(22, 64);
q.swap();
System.out.println(q.x − q.y);
```

Example 2.3 Homogenous translation of polymorphism into Java

```
class Pair {
    Object x;  Object y;
    Pair (Object x, Object y) {this.x = x; this.y = y;}
    void swap () {Object t = x; x = y; y = t;}
}

class Integer {
    int i;
    Integer (int i) { this.i = i; }
    int intValue() { return i; }
}

Pair p = new Pair((Object)"world!", (Object)"Hello,");
p.swap();
System.out.println((String)p.x + (String)p.y);

Pair q = new Pair((Object)new Integer(22),
                  (Object)new Integer(64));
q.swap();
System.out.println(((Integer)(q.x)).intValue() −
                  ((Integer)(q.y)).intValue());
```

Igarashi, Pierce, and Wadler (1999) — Featherweight Java

$$\Gamma \vdash \mathtt{x} : \Gamma(\mathtt{x})$$

$$\frac{\Gamma \vdash \mathtt{e}_0 : \mathtt{C}_0 \qquad \mathit{fields}(\mathtt{C}_0) = \overline{\mathtt{C}}\ \overline{\mathtt{f}}}{\Gamma \vdash \mathtt{e}_0.\mathtt{f}_i : \mathtt{C}_i}$$

$$\frac{\Gamma \vdash \mathtt{e}_0 : \mathtt{C}_0 \qquad \mathit{mtype}(\mathtt{m}, \mathtt{C}_0) = \overline{\mathtt{D}} \rightarrow \mathtt{C} \qquad \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{C}} \qquad \overline{\mathtt{C}} <: \overline{\mathtt{D}}}{\Gamma \vdash \mathtt{e}_0.\mathtt{m}(\overline{\mathtt{e}}) : \mathtt{C}}$$

$$\frac{\mathit{fields}(\mathtt{C}) = \overline{\mathtt{D}}\ \overline{\mathtt{f}} \qquad \Gamma \vdash \overline{\mathtt{e}} : \overline{\mathtt{C}} \qquad \overline{\mathtt{C}} <: \overline{\mathtt{D}}}{\Gamma \vdash \mathtt{new}\ \mathtt{C}(\overline{\mathtt{e}}) : \mathtt{C}}$$

$$\frac{\Gamma \vdash \mathtt{e}_0 : \mathtt{D} \qquad \mathtt{D} <: \mathtt{C}}{\Gamma \vdash (\mathtt{C})\mathtt{e}_0 : \mathtt{C}}$$

$$\frac{\Gamma \vdash \mathtt{e}_0 : \mathtt{D} \qquad \mathtt{C} <: \mathtt{D} \qquad \mathtt{C} \neq \mathtt{D}}{\Gamma \vdash (\mathtt{C})\mathtt{e}_0 : \mathtt{C}}$$

$$\frac{\Gamma \vdash \mathtt{e}_0 : \mathtt{D} \qquad \mathtt{C} \not<: \mathtt{D} \qquad \mathtt{D} \not<: \mathtt{C} \qquad \mathit{stupid\ warning}}{\Gamma \vdash (\mathtt{C})\mathtt{e}_0 : \mathtt{C}}$$

Igarashi, Pierce, and Wadler (1999)
— Featherweight Generic Java

$$\Delta;\Gamma \vdash \mathtt{x} : \Gamma(\mathtt{x})$$

$$\frac{\Delta;\Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \mathit{fields}(\mathit{bound}_\Delta(\mathtt{T}_0)) = \bar{\mathtt{T}}\ \bar{\mathtt{f}}}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{f}_i : \mathtt{T}_i}$$

$$\frac{\begin{array}{c}\Delta;\Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \mathit{mtype}(\mathtt{m}, \mathit{bound}_\Delta(\mathtt{T}_0)) = \mathtt{<}\bar{\mathtt{Y}} \triangleleft \bar{\mathtt{P}}\mathtt{>}\bar{\mathtt{U}} \to \mathtt{U} \\ \Delta \vdash \bar{\mathtt{V}}\ \mathrm{ok} \qquad \Delta \vdash \bar{\mathtt{V}} <: [\bar{\mathtt{V}}/\bar{\mathtt{Y}}]\bar{\mathtt{P}} \qquad \Delta;\Gamma \vdash \bar{\mathtt{e}} : \bar{\mathtt{S}} \qquad \Delta \vdash \bar{\mathtt{S}} <: [\bar{\mathtt{V}}/\bar{\mathtt{Y}}]\bar{\mathtt{U}}\end{array}}{\Delta;\Gamma \vdash \mathtt{e}_0.\mathtt{m}\mathtt{<}\bar{\mathtt{V}}\mathtt{>}(\bar{\mathtt{e}}) : [\bar{\mathtt{V}}/\bar{\mathtt{Y}}]\mathtt{U}}$$

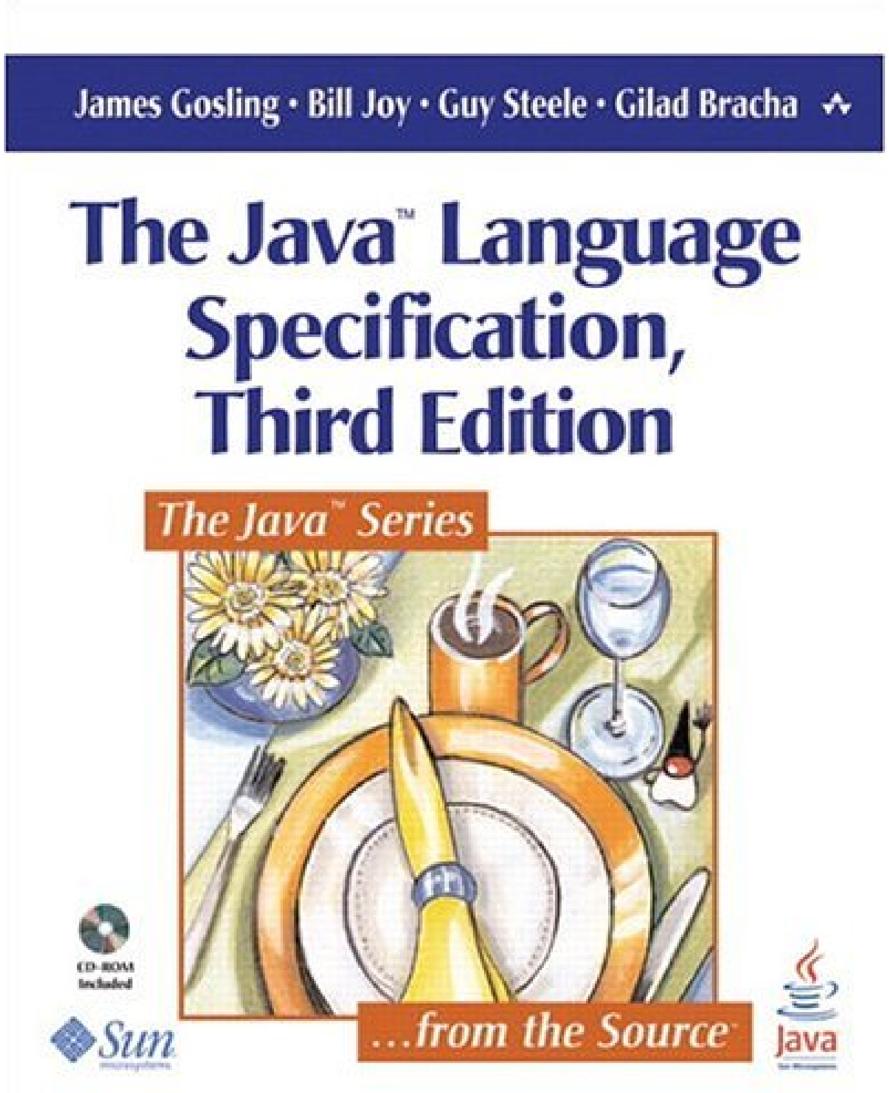
$$\frac{\Delta \vdash \mathtt{N}\ \mathrm{ok} \qquad \mathit{fields}(\mathtt{N}) = \bar{\mathtt{T}}\ \bar{\mathtt{f}} \qquad \Delta;\Gamma \vdash \bar{\mathtt{e}} : \bar{\mathtt{S}} \qquad \Delta \vdash \bar{\mathtt{S}} <: \bar{\mathtt{T}}}{\Delta;\Gamma \vdash \mathtt{new}\ \mathtt{N}(\bar{\mathtt{e}}) : \mathtt{N}}$$

$$\frac{\Delta;\Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta \vdash \mathit{bound}_\Delta(\mathtt{T}_0) <: \mathtt{N}}{\Delta;\Gamma \vdash (\mathtt{N})\mathtt{e}_0 : \mathtt{N}}$$

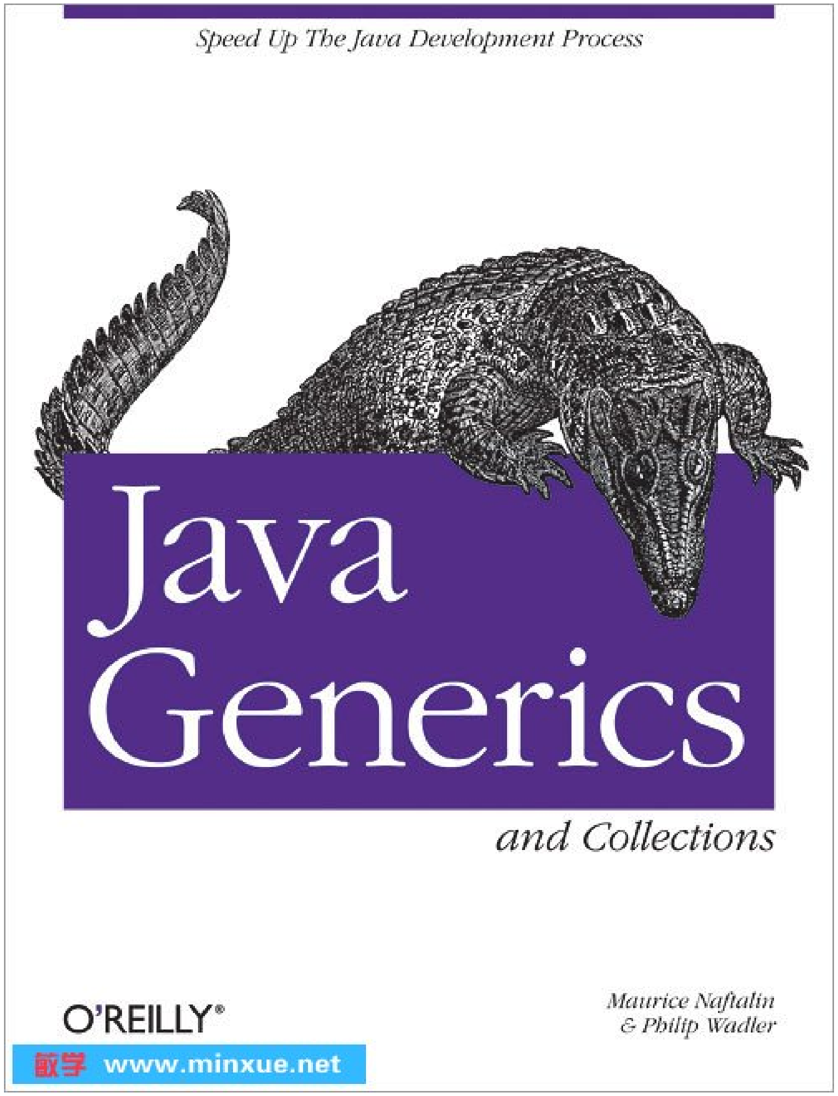
$$\frac{\begin{array}{c}\Delta;\Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta \vdash \mathtt{N}\ \mathrm{ok} \qquad \Delta \vdash \mathtt{N} <: \mathit{bound}_\Delta(\mathtt{T}_0) \\ \mathtt{N} = \mathtt{C}\mathtt{<}\bar{\mathtt{T}}\mathtt{>} \qquad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{D}\mathtt{<}\bar{\mathtt{U}}\mathtt{>} \qquad \mathit{dcast}(\mathtt{C}, \mathtt{D})\end{array}}{\Delta;\Gamma \vdash (\mathtt{N})\mathtt{e}_0 : \mathtt{N}}$$

$$\frac{\begin{array}{c}\Delta;\Gamma \vdash \mathtt{e}_0 : \mathtt{T}_0 \qquad \Delta \vdash \mathtt{N}\ \mathrm{ok} \qquad \mathtt{N} = \mathtt{C}\mathtt{<}\bar{\mathtt{T}}\mathtt{>} \qquad \mathit{bound}_\Delta(\mathtt{T}_0) = \mathtt{D}\mathtt{<}\bar{\mathtt{U}}\mathtt{>} \\ \mathtt{C} \not\trianglelefteq \mathtt{D} \qquad \mathtt{D} \not\trianglelefteq \mathtt{C} \qquad \mathit{stupid\ warning}\end{array}}{\Delta;\Gamma \vdash (\mathtt{N})\mathtt{e}_0 : \mathtt{N}}$$

Gosling, Joy, Steele, Bracha (2004) — Java 5



Naftalin and Wadler (2006)



Part III

Haskell: Type Classes

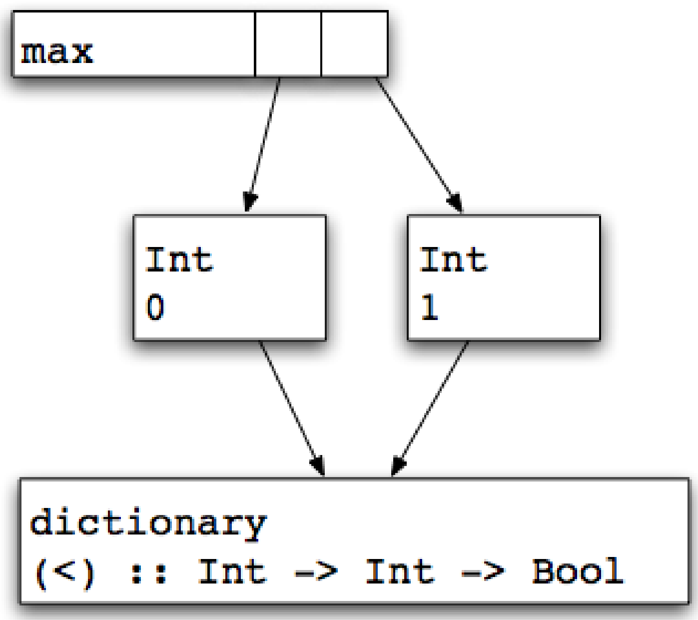
Type classes

```
class Ord a where
  (<) :: a -> a -> Bool

instance Ord Int where
  (<) = primitiveLessInt

instance Ord Char where
  (<) = primitiveLessChar

max  :: Ord a => a -> a -> a
max x y | x < y       =  y
        | otherwise   =  x

maximum :: Ord a => [a] -> a
maximum [x]      =  x
maximum (x:xs)   =  max x (maximum xs)

maximum [0,1,2] == 2
maximum "abc" == 'c'
```

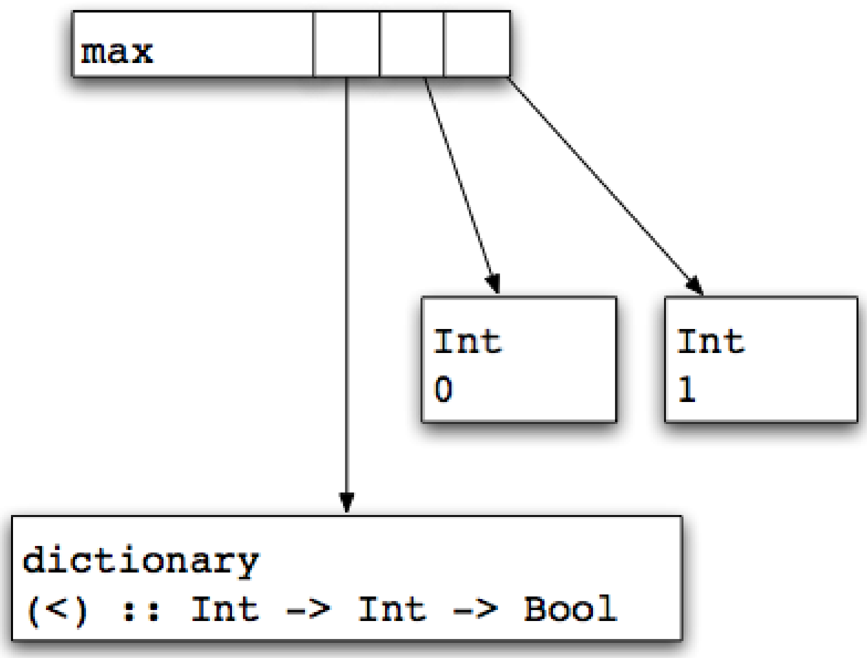
Translation

```
data Ord a = Ord { less :: a -> a -> Bool }

ordInt :: Ord Int
ordInt =  Ord { less = primitiveLessInt }

ordChar :: Ord Char
ordChar =  Ord { less = primitiveLessChar }

max  :: Ord a -> a -> a -> a
max d x y | less d x y  =  x
          | otherwise   =  y

maximum :: Ord a -> [a] -> a
maximum d [x]      =  x
maximum d (x:xs)   =  max d x (maximum d xs)

maximum ordInt [0,1,2] == 2
maximum ordChar "abc" == 'c'
```

Object-oriented



Type classes



Type classes, continued

```
instance Ord a => Ord [a] where
  [] < []                        =  False
  [] < y:ys                      =  True
  x:xs < []                      =  False
  x:xs < y:ys | x < y            =  True
              | y < x            =  False
              | otherwise        =  xs < ys

maximum ["zero","one","two"] == "zero"
maximum [[[0],[1]],[[0,1]]] == [[0,1]]
```

Translation, continued

```
ordList :: Ord a -> Ord [a]
ordList d  =  Ord { less = lt }
  where
  lt d [] []                            =  False
  lt d [] (y:ys)                        =  True
  lt d (x:xs) []                        =  False
  lt d (x:xs) (y:ys) | less d x y  =  True
                     | less d y x  =  False
                     | otherwise   =  lt d xs ys

maximum d0 ["zero","one","two"] == "zero"
maximum d1 [[[0],[1]],[[0,1]]] == [[0,1]]
  where
  d0 = ordList ordChar
  d1 = ordList (ordList ordInt)
```

Maximum of a list, in Java

```
public static <T extends Comparable<T>>
  T maximum(List<T> elts)
{
  T candidate = elts.get(0);
  for (T elt : elts) {
    if (candidate.compareTo(elt) < 0) candidate = elt;
  }
  return candidate;
}

List<Integer> ints = Arrays.asList(0,1,2);
assert maximum(ints) == 2;

List<String> strs = Arrays.asList("zero","one","two");
assert maximum(strs).equals("zero");

List<Number> nums = Arrays.asList(0,1,2,3.14);
assert maximum(nums) == 3.14;  // compile-time error
```

Part IV

Three recent ideas

Idea I: Blame calculus

$$
\begin{array}{rcll}
v : A{\to}B \Rightarrow^{p} A'{\to}B' & \longrightarrow & \lambda x'{:}A'.\,(v\;(x' : A' \Rightarrow^{\overline{p}} A) : B \Rightarrow^{p} B') & \\
v : G \Rightarrow^{p} G & \longrightarrow & v & \text{if } G \neq \star{\to}\star \\
v : A \Rightarrow^{p} \star & \longrightarrow & v : A \Rightarrow^{p} G \Rightarrow \star & \text{if } A \prec G \text{ and } A \neq \star \\
v : G \Rightarrow \star \Rightarrow^{p} A & \longrightarrow & v : G \Rightarrow^{p} A & \text{if } G \prec A \\
v : G \Rightarrow \star \Rightarrow^{p} A & \longrightarrow & \texttt{blame}\; p & \text{if } G \not\prec A \\
(v : G \Rightarrow \star)\;\texttt{is}\; G & \longrightarrow & \texttt{true} & \\
(v : H \Rightarrow \star)\;\texttt{is}\; G & \longrightarrow & \texttt{false} & \text{if } G \neq H \\
E[\texttt{blame}\; p] & \longmapsto & \texttt{blame}\; p & \text{if } E \neq [\cdot]
\end{array}
$$

Idea II: Propositions as Sessions

$$\frac{}{w \leftrightarrow x \vdash w : A^{\perp}, x : A}\ \mathsf{Ax} \qquad \frac{P \vdash \Gamma, x : A \quad Q \vdash \Delta, x : A^{\perp}}{\nu x : A.(P \mid Q) \vdash \Gamma, \Delta}\ \mathsf{Cut}$$

$$\frac{P \vdash \Gamma, y : A \quad Q \vdash \Delta, x : B}{x[y].(P \mid Q) \vdash \Gamma, \Delta, x : A \otimes B}\ \otimes \qquad \frac{R \vdash \Theta, y : A, x : B}{x(y).R \vdash \Theta, x : A \parr B}\ \parr$$

$$\frac{P \vdash \Gamma, x : A}{x[\mathsf{inl}].P \vdash \Gamma, x : A \oplus B}\ \oplus_1 \qquad \frac{P \vdash \Gamma, x : B}{x[\mathsf{inr}].P \vdash \Gamma, x : A \oplus B}\ \oplus_2 \qquad \frac{Q \vdash \Delta, x : A \quad R \vdash \Delta, x : B}{x.\mathsf{case}(Q, R) \vdash \Delta, x : A \,\&\, B}\ \&$$

$$\frac{P \vdash ?\Gamma, y : A}{!x(y).P \vdash ?\Gamma, x : !A}\ ! \qquad \frac{Q \vdash \Delta, y : A}{?x[y].Q \vdash \Delta, x : ?A}\ ?$$

$$\frac{Q \vdash \Delta}{Q \vdash \Delta, x : ?A}\ \mathsf{Weaken} \qquad \frac{Q \vdash \Delta, x : ?A, x' : ?A}{Q\{x/x'\} \vdash \Delta, x : ?A}\ \mathsf{Contract}$$

Idea III: Object to Object

- Object vitiates parametricity

```
class Object {
  Bool eq(Object that) {...}
  String show() {...}
}
```

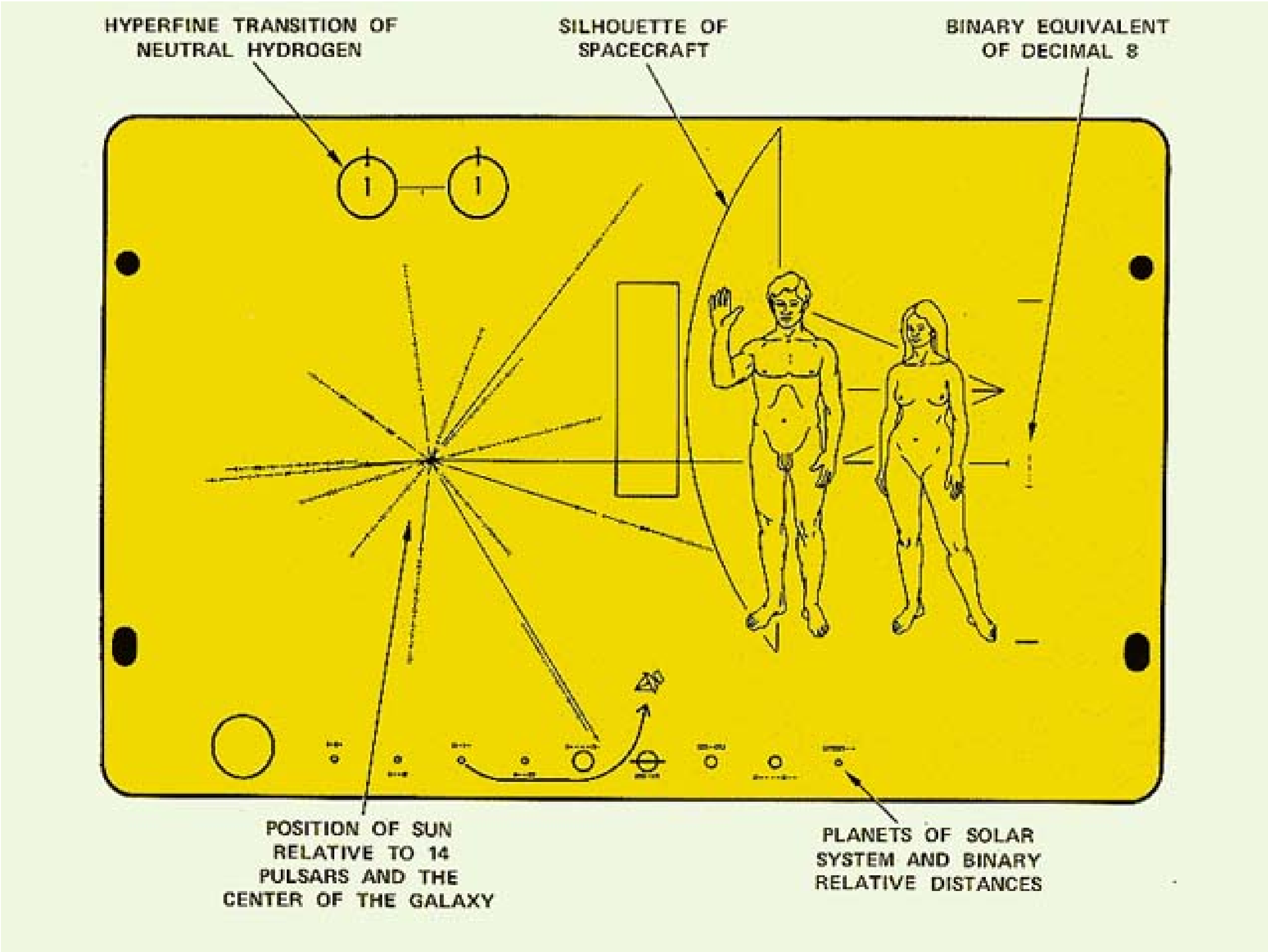
- Top preserves parametricty

```
class Object {
  // no methods!
}
```

Part V

Aliens

How to talk to aliens



Independence Day

A universal programming language?

Lambda is Omniversal